Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE UNITS OF MEASURE AND THE PRINCIPLE OF RELATIVITY.

> "Mathematica, quae philosophiam naturalem terminare, non generare aut procreare."—Bacon.

THE ideas which underlie scientific hypothesis, however diverse they may seem to be, rest on a common mechanical base. All phenomena are therefore explained as being due to some special form of mechanical motions of bodies, and can be reduced ultimately to modifications of inertia, velocity, momentum, or energy. These hypotheses have, in addition, this common property: they begin with the postulate of an entity which possesses inertia and velocity. Various names have been attached to this entity, but whether it be called an atom, an ether, a vortex, electricity, or energy, its function is the same. While there is little difference in the nature of this entity or substance, yet there are two diametrically opposite ways of considering its space attributes which give rise to two irreconcilable schools of thought. One of these believes the primordial substance to be discontinuous, with its parts separated by vacuous space, and the other school just as confidently maintains that there *can be no empty* space and that substance is continuous. Arguments for and against these two views have been unceasing and are likely to continue so long as hypothetical methods remain in vogue. The problem is quite incapable of solution because the postulate of continuity or discontinuity is a matter of conviction rather than of reason. The fact is, we believe one or the other to be true and that is the end of it. In such discussions where a postulate is quite beyond our powers of verification by experience, each contestant chooses his starting point, and this once chosen, his argument in favor of it is as sound as that of his opponents. And the truth of this assertion is evident, because from these diametrically opposite postulates, conclusions which are quite identical are deduced by equally logical arguments; witness the controversy between the atomists and the Cartesians which see-saws through all scientific theory and is as far from settlement to-day as it was centuries ago. This dualism, as it may be called, is not confined to science; it is but one phase of that larger dualism of philosphy and ethics which has come down to us from the Greek thinkers; the claims of two opposing schools of thought are always present to show us that, if the mind can develop a system from one set of postulates, the same results can be obtained from their contraries.

If then our conception of the nature of things is but a matter of personal conviction, what value is there in the opinion of Professor Lorentz, that, if we wish to obtain an insight into the mysteries and operations of nature, we must make hypotheses? If such hypotheses are not capable of proof then they must rest on what Bolingbroke so aptly calls our inward sentiment of knowledge. The experimentalist who described phenomena from his inward sentiment of what they should be, rather than from observations of what they are, would be classed as a nuisance. Such a one not only does not advance our knowledge but he actually retards it, since his work must be repeated before the truth can be known. So, too, the theorist who relies on hypotheses and not on facts and laws builds a structure which is not only temporary and false but must be torn down. If we relegate to metaphysics, so large a part of what is commonly called physics, what is left to the science of physics? The answer probably is to be found in the saying of Lord Kelvin, that no scientific statement is understood until it is measured. If we grant this, then we must examine, in such a discussion as this, our methods of measurement.

The statement of a phenomenon invariably contains two terms, called qualitative and quantitative factors; the former expressing "what kind" and the latter, "how much." Thus, when we speak of a distance as ten centimeters, we mean that we are to consider a quantity ten of the quality, length. Now it is not really the province of science to seek for absolute knowledge of either qualities or quantities, and the attempt to do so is the excuse for hypothesis. What science is concerned with, is the relative knowledge, or comparison, of different quantities of any given quality and the reduction of complex qualities into combinations of simpler ones.

It thus becomes of prime importance to settle on the simplest and most fundamental qualities which may serve as a foundation for our system of measurement. Because of the fact that mechanical motions and mechanical forces are the most readily perceived by us and are most easily expressed in mathematical formulae, the fundamental units of quality are always selected from mechanical concepts. Of these, length, time, and mass, are found to be incapable of further simplification and have been adopted as the units of measurement. As these qualities must enter into all equations quantitatively, and as we can have no conception of an absolute quantity, we have by statute fixed upon certain arbitrary standards of quantity, such as the centimeter and foot for length, the gram and pound for mass, the second for time. From these prime standards all other quantities of a mechanical nature may be readily derived;

as, for example, velocity is the quotient of a given length by a time; momentum is the product of a mass and its velocity; energy, one-half the product of a mass and a square of its velocity, etc.

The science of mechanics is the only branch of physics which has a completely developed theory. Not only is this the case, but the other branches of physics have been developed from a mechanical basis, in so far at least as the measurement of all phenomena is now made in terms of these mechanical units. This coordination in physics has the great advantage of making it possible to express what are apparently unrelated phenomena by combinations of only three standards of measurement and by a few general equations of mechanics, and we attain the additional advantage of expressing all these phenomena in the most concrete form imaginable. But, on the other hand, this method produces a serious break between physics and certain other sciences, especially those which rely more exclusively on the sense-perceptions as criteria for classification of phenomena. Thus, to the physicist light of different kinds is distinguished only by the three qualities possible in a mechanical wave; namely, its wave-length, the amplitude of its disturbance, and the complexity of its form. These same qualities must also serve to distinguish sounds, water waves and many other types of this kind of motion. For the biologist and psychologist, light is distinguished by its tint, intensity, and saturation, the three qualities which affect the sense of sight; and sound is determined by pitch, loudness, and timbre, which affect the auditory nerve. As there is no relation between the sensations of sight and hearing, so there can be no connection between light and sound. While the method of the physicist has the great advantage of unity, yet it suffers from its artificiality, as it tends to bring into undue prominence the mechanical energy involved in producing light and sound and ignores the more important properties of these phenomena, their effect on our senses.

It is generally conceded that we have no abstract or absolute knowledge of a quantity of length, time, or mass. In other words, we can express any of these three quantities only as a numerical ratio with respect to a predetermined standard of the same quantity, such as a yardstick for length, a pound for mass, and a given motion of a clock-hand or of the rotation of the earth for time; and from this fact, it is often argued that we cannot form any idea of the qualities, space, time, and matter, except as they are individually and concretely measured. Kant, to be sure, maintained that we were endowed with an innate and inexplicable, but sufficient, idea of pure space and time. These qualities are, however, by themselves inappreciable to our senses. To make them sensible, we need a third, which he calls the *Ding an sich,* corresponding in the external world to what we call the entity, matter.

However vulnerable and unsatisfactory the doctrine of innate ideas may be, yet it seems to me that by the abstractive method as used in the science of mechanics we obtain a real and adequate idea of these three fundamental postulates. If we have not such a power of abstracting real ideas from our concrete observations of phenomena, then it is difficult to believe that the conclusions of Euclidean geometry and of mathematical analysis in general are rigorous. Thus, in mathematical definitions, space is a mere volume enclosed in an imagined boundary which may be conceived as of any extent from the indefinitely small to the indefinitely large. And this imagined bounding surface bears no closer relation to a concrete material envelope than does an image in a mirror to its object. It seems to me that I have an adequate idea, for instance, of the space in an empty room and that I can abstract all the properties from my concrete perception of the material walls of this room except the one fact that they enclose this space. By deduction and by experience I am convinced that this space is not empty, but contains air. But it is really more difficult to appreciate the existence of the air, if it be at rest, than its non-existence. This belief is borne out by the slow advance in the conception of the existence of gases. And it also seems true to me that I have an adequate idea of an area enclosed in a triangle, because I can think of the area enclosed by three abstract lines and refrain from thinking of the concrete volume of any real lines drawn with a pencil.

As for the concept time, we have a twofold sense of it; one is the coincidence of an event with the position of the hands of a clock or of the earth. But we have, in addition, a much more general idea of time, as mere succession of events without any reference to such a standard of measurement as the second or minute. This might be called our belief that events entirely unconnected with our own experience occur successively just as those events do which we compare with a clock. We have, on awaking from sleep, a distinct and clear idea that events have transpired successively during our unconsciousness and without any reference to a measured interval of time. This general knowledge of "before and after" is apparently possible to an animal; a dog will with certainty expect a reward after the performance of a trick and not before or during it.

As for the third postulate, which is called matter, it may be taken in the most general sense as the something which makes space and time concrete to us. The necessity in science for such a postulate is clear, for science deals with the phenomena of an objective world. Even those men of science most opposed to the idea that mass is the measure of matter, still postulate an entity identical with matter, as just defined, although they may call it by a different name, such as energy, electricity, etc. The distinction between mathematics and science lies in the conception of this third postulate. In pure mathematics matter becomes the abstract postulate, quantity. When discussing velocity abstractly there is then no need to consider the properties of the things moving, but in physics there is such a need. Or again, when dealing with space relations the mathematician is in no way limited to the restricted number of three dimensions which our sense-perception of the material universe imposes on science.

The supreme value of mathematics to science is due to the fact that scientific laws and theories have their best, if not their only complete, expression in mathematical formulae; and the degree of accuracy with which we can express scientific theory in mathematical terms is the measure of the state of a science. Thus it is possible to classify sciences according to their development, from the accumulation of statistics of phenomena to the generalization of these phenomena in comprehensive and rigorous laws. In such a classification, sociology or the study of existing society occupies the lowest rank, since true laws can be derived only from actions whose completed consequences are known. Sociology therefore attempts to found its laws on the data of history, the study of past society; history must in the same way rely on psychology, which deals with the actions of the individuals of society; psychology relies on biology; biology, on chemistry; chemistry, on physics; and physics on pure mathematics. While each science thus strives to found its laws on the conclusions of the following science, each succeeds only partially; this leads us to a paradox. The goal of science is mathematics, and while mathematics may be said to be the only true science since it has the only true scientific method, mathematics is no science because it deals with abstractions and ignores concrete phenomena.

As stated before, all quantities in mechanics may be expressed by combining the fundamental units—length, mass, and time—in simple ratios and products. And while the phenomena of light, heat, and electricity do not manifest themselves directly to our senses in any manner which may be coordinated with mechanical actions, yet we endeavor to measure them in physics as if they were mechanical in nature. Thus we speak of the velocity of light from the sun to the earth, as if we were considering a real mechanical motion of a ponderable body, although in the intervening space there is no matter. The only thing we have been able to observe is that light emitted from the sun appears on the earth some time later. Light as a phenomenon does not exist unless it is associated with matter; we can no more discuss the amount of light or its velocity in absolutely vacuous space than we can speak of the temperature of such space. Light has a true mechanical velocity when passing through space occupied by matter, for then we can observe and measure its path as well as the time function and so obtain a value for velocity. From observations on the velocity of light in matter and from analogy to the phenomena of sound transmission, which takes place only in spaces occupied by matter, we by the hypothetical method transfer the measurements and laws of light in transparent bodies to space not occupied by matter. The very assumption of a light-velocity in immaterial spaces requires us also tacitly to assume that *something,* in a mechanical sense, is moving. Once we have granted that light is something moving, then all the other phenomena of light permit of a hypothetical mechanical explanation, and we have the right to speak of the momentum and energy of this light something, whether it be corpuscles or waves. But it is the easiest thing in the world to forget that we can never obtain any real knowledge of this something we have called light. It is surprising that men of science should believe that they have proved the existence of the ether, as when Sir Oliver Lodge states that it is the most massive thing imaginable; or that space is occupied by energy, an entity possessing inertia and probably gravitational force as Professor Einstein announces: they should see that such statements are not deductions made from our experimental knowledge of light, but are already contained in the hypothetical postulate that light is mechanical and has a mechanical velocity.

It is instructive to consider, in this connection, how we also have attached hypothetically the science of electricity to mechanics. The fundamental phenomenon observed, when bodies are electrified or magnetized, is that they attract or repel each other with a mechanical force which, like the force of gravitation, varies inversely as the square of the distance between them. In our fundamental units this mechanical force is equal to a mass times a length and divided by the square of a time. Now Coulomb, who discovered the law of electrical attraction, believed that electricity was a kind of fluid substance, such as was always introduced when phenomena were obscure; and with this idea in his mind, he employed the term "quantity" of electricity to indicate an analogy with a quantity of matter. On this supposition, a quantity of electricity expressed in mechanical units is equal to the square root of a length times a mass. Now it is quite certain that a quantity of electricity has nothing in common with length. Taking a step further, we find that the resistance of a conductor to an electrical current must be expressed as a velocity. Yet it would be absurd to attach any concrete relation between electrical resistance and mechanical velocity.

These three examples of velocity illustrate quite clearly the difference between the abstractive and hypothetical methods. From the definite and clear idea of the mechanical velocity of a concrete body, we pass to an indefinite idea of the velocity of light in empty space, and in order to link this idea to mechanical notions, we assign to light a hypothetical material existence. But our mathematical equations lead us a step further and we can derive a purely formular relation between mechanical velocity and electrical resistance; in this case analogy between physics and mathematics entirely fails and no idea, even hypothetical, has been attached to the result.

The principles of the science of mechanics, on which the theories of the other branches have been built, date from the time of Galileo and Newton. As they had withstood the searching criticism of the masters of the science for centuries, without having been shown to be either false or inadequate, the belief grew that however other theories of physics might change, the laws of mechanics as stated by Newton were probably final. But it is inevitable that if we found the laws of light and electricity on mechanics, the time will come when the accumulation of knowledge will increase the discrepancies which must always exist between any two branches of science and will eventually require a thorough revision of one or the other. If the attention be directed more toward discovering the phenomena and laws of light and electricity than of mechanics, as it is to-day, any discrepancies will probably be laid to the laws of mechanics and their revision will be attempted to ensure agreement. This has occurred in the last few years, and the mechanics based on material bodies is being replaced by a mechanics of electricity or by one of energy, if the terms be permissible.

The mechanics of material bodies, to which the name of Newton is generally attached, was based on the objective reality of matter, whose quantitative measure was inertia or mass. Newton evidently considered inertia as a fundamental attribute of matter, and thus invariable and inexplicable; something to be accepted and determined solely by experiment. Thus he says in his *Principia*: "Haec (materiae vis) semper proportionalis est suo corpore, neque differt quicquam ab inertia massae, nisi in modo concipiendi. Per inertiam materiae fit, ut corpus omne de statu suo vel quiescendi vel movendi difficulter deterbetur." This postulate may be freely translated to mean that the force of attraction of matter is always proportional to the amount of matter acting, and does not differ in any way from the inertia of mass except in our method of apprehending it. By inertia of matter is meant, that a body can be changed from its previous condition of rest or motion only by this material force. It is evident that he regarded inertia as an inherent and inalienable property of a body, independent of the influence of any other body or ether, and forming a connecting and stable link between ourselves and the external world. Such being the case, how may we decide what is the mass of any particular body? If a number of individuals measure experimentally a mass, or even if one of them measures it several times, no two observations will agree. Which observation gives the correct value? Newton would have answered, none of them. Data of objective phenomena can never be known exactly; each value we obtain approximates to the truth, and the approximation is the closer, the greater the number so obtained and the greater the care exercised. The final result must be deduced from all the observations, according to a well-developed mathematical theory of errors. The same reasoning was held to apply to observations on the space-dimensions of a body and on the time occurring during any event.

While the idea was advanced in this mechanics that the position and motion of any body could be determined only from the position of some other body, supposed for the time being to be at rest, or that position and motion were relative and not absolute; yet it was not explicitly stated that there could not be absolute rest or motion, such as would occur if the motion of a body were referred to an absolutely fixed center of the universe or to an ether which was incapable of motion. For all practical problems, Newton's third law of motion, which states that to every action there is an equal and oppositely directed reaction, announces the universality of relativity.

We should finally note that mass, dimensions, and time were held to be unaffected by the motion of a body. Newton expresses this by saying that force-actions of matter, or the science of dynamics, are independent of its initial state of rest or motion.

The first serious criticism of these postulates was made by a number of physicists, forming what is often called the school of energetics, who proposed to substitute energy for mass as the fundamental attribute of matter. At the time, the change was rather immaterial as we were accustomed to think that mass and energy were coexistent and that either one was unintelligible without the other. It was the same kind of a problem as deciding which came first, the owl or the egg; the answer to which is that we know nothing about the matter.

The discrepancies in mechanics did not prove to be embarrassing until certain problems connected with light and electricity became pressing. Of these, three stand out most prominently.

The phenomena associated with electricity, when it passes through very high vacua, and with radioactive bodies like radium, are now explained as being due to the action of excessively small particles, carrying a charge of electricity and moving with a velocity comparable to the velocity of light, or about one hundred thousand miles per second. These particles, both because of their smallness and because of their velocity, are in an entirely different class from the bodies previously considered in mechanics which have a sensible mass and whose greatest velocity is less than one hundred miles per second. Errors in mechanical laws which would otherwise be inappreciable may easily assume large proportions when applied to such extreme cases. The hypothesis of the electron has profoundly modified the conception of the atom, and the nature of matter and motion.

In the second place, recent experiments lead to the belief that matter is always electrified; that light is due to electrical variations; and that the amount of electricity associated with a given amount of matter is a fixed and invariable quantity. It was further shown by theory and confirmed by experiment that if an electrified body be moved, the ratio of its electrical charge to its mass must vary with the velocity of its motion; and since the electrical charge is held to be a constant, there remains only the alternative of considering the mass of a body as a variable. This evidently strikes at the very root of Newtonian mechanics. The effect of motion on mass is found to be inappreciable until the velocity approximates to that of light, so the discussion would have remained a purely academic one if the discovery of the electron had not brought us suddenly face to face with bodies which are supposed to have a velocity great enough to affect experimentally their mass. A further consequence of this theory is that mass becomes infinite when the velocity of light is reached. While this velocity has always been considered enormous and beyond our power to attain, such a result was entirely unsuspected. The laws of dynamics of bodies at rest could evidently no longer be held to be the same as those for bodies in motion.

Lastly, various experiments have been made to find an effect due to the mutual relations of the ether and matter. None could be found. One in particular, devised by

Professors Michelson and Morley, has proved to be the hardest problem in modern physics to explain. Calculation showed that certain properties of light should be affected appreciably by the motion of the earth through space, but the experiment proved beyond doubt that such was not the case. The first attempt at an explanation was made by assigning certain complicated motions to the ether which would annul the effect the motion of the earth should produce; but it was shown that unless the ether remained absolutely at rest worse difficulties resulted. Then the bold assumption was made, often called the Fitzgerald-Lorentz effect, that the dimensions of bodies in motion were so changed by their motion as to neutralize the effect of the earth's motion on the phenomena of light. Every body would, according to this idea grow shorter, the faster it moved; and would flatten out to nothing if it could attain a speed equal to the velocity of light.

One of the results of this criticism of mechanics has been to change profoundly our ideas of the ether. As is well known, the early conception of the ether was a kind of material substance possessing properties incompatible with those of any other kind of matter. A large part of the effort of theorists down to the middle of the nineteenth century was devoted to inventing attributes for the ether which would enable it to fulfil its function as the medium for the transmission of light. At this time, Faraday discovered an effect in electricity which required an entire change in our ideas of the ether. When he found that static electric charges and forces were dependent on the characteristics of ether, and when later Hertz showed experimentally that electromagnetic energy passed through the ether, and that this form of radiant energy was undoubtedly of the same type as light and heat; then it was found that no material ether could be imagined which would perform these new duties, especially as it was already staggering under the burdens of the old ones. Faraday proposed as a substitute that we should no longer imagine the ether to be a substance having material or mechanical properties, but one responding to electrical and magnetic actions. He was far in advance of his time and to be acceptable the hypothesis needed the mathematical development which was so skilfully accomplished by Maxwell. Both Faraday and Maxwell, while really destroying the material nature of the ether, strove to maintain at least a partial connection between electro-magnetic and mechanical attributes. For this reason they supposed that electro-magnetic stresses manifested themselves by creating mechanical strains in the ether and in matter immersed in it. Such strains must produce actual physical deformations of size and shape in all electrified bodies. It has been shown experimentally by the writer that such deformations are not produced in electrified matter and lately Professor Lorentz has stated, that since Helmholtz proved these stresses would cause the ether to move, and since no experiment has ever shown us a trace of a motion in the ether, we must deny the real existence of ether stresses. The effect of such a denial is to separate all attributes of the ether from concrete realities and to class them as abstract symbols. Professor Lorentz is quite explicit on this point, as he holds that, "while thus denying the real existence of ether stresses, we can still avail ourselves of all the mathematical transformations by which the application of the formula (for these stresses) may be made easier: ....and for convenience's sake we may continue to apply to the quantities occurring in this integral the name of stresses. Only, we must be aware that they are only imaginary ones, nothing else than auxiliary mathematical quantities." The dematerialization of the ether, when it was once found that light and heat still managed to come to us from the sun although its medium of transmission had received such a rude shock, has progressed rapidly until to-day many accept the postulate that there is no difference between absolutely vacuous space and the ether except that the latter is the temporary seat of radiant energy and possesses a light vector. If this last definition of the ether means anything in a concrete sense, it implies that light has the power of changing a vacuum into an ether. When we stop to think that a vacuum means negation of everything, we realize what extraordinary things light and the ether are in modern physics. Just consider this statement of Professor Einstein, which is considered almost authoritative, "the places in space where these electro-magnetic actions (i. e., light) occur are here considered not as states of a sort of matter, but as self-existing things which are similar to ponderable matter and in common with it have the characteristic of inertia." One has merely to ask, what becomes of these self-existing things when light ceases to go through a certain space and it slips back into its state of vacuity?

Without going into details, we find a like trend in the hypothesis of the nature of matter. It began with the atom, as a minute simulacrum of ponderable bodies and then we proceeded to strip away one concrete attribute after another until for a brief interval matter was a manifestation of an entity, electricity. But even this idea was too concrete, too material, to serve and now the objective universe is the symbol, energy.

If we finally subscribe to these ideas, are we not really acknowledging that hypothetical science has failed as the interpreter of an objective world? It seems to me the guides to knowledge are now to be found in those subjective impressions which must depend on the individual and vary with him. Scientific laws are thus not facts to be discovered but the temporary consensus of opinion of a number of individuals who, for the time being, find themselves in agreement. The classical natural philosophy of Newton and Galileo has drifted into the transcendental symbolism which is apt to occur when German thinkers become the leaders in philosophy.

Evidently an almost chaotic condition had developed in our ideas of space, matter, and electricity. Each theorist advanced an hypothesis of the most tentative character which all felt to be inadequate. Yet in spite of the confusion, three points of agreement may be noted. In the first place, the ether as a material substance was impossible and even useless; the name, it is true, was left but it expressed now but the shadow of a reality. When the ether is called an electro-magnetic something, or a space differentiated only by the transient presence of energy, the name may signify something but it is hardly intelligible except as a mathematical symbol. Secondly, these theories agree in assuming electricity to be an entity; that is, we are to suppose that it is not due to a state of matter manifested by a special mechanical force, but a substance like matter, divisible into atomic elements and having inertia. Thirdly, the mass inertia of matter is not an invariable quantity but changes with the velocity of a body; thus it is the measure of matter only while there is no motion.

Such was the state of physical theory when Professor Einstein announced, in an article published in the *Annalen der Physik* for the year 1905, a new scientific principle which he believes will reconcile the contradictions of our new hypotheses; satisfy the three conceptions mentioned; and correct and amplify the Newtonian mechanics so that it will again harmonize with modern science. Whether this "principle of relativity" will accomplish all that its discoverer hopes can be decided only after an exhaustive trial. But there is no doubt as to the startling effect it has produced on scientific theory nor of the far-reaching importance of its conclusions. For example, Professor

Planck is said to have declared that this new conception of time and space surpasses in boldness anything that has appeared up to the present time in speculative science.

The name, "principle of relativity," is derived from Professor Einstein's first postulate, which is as follows: The idea of absolute rest or absolute motion is an impossibility to the human mind, and not only in mechanics, but also in electrodynamics corresponds to no properties of phenomena.

In addition, he announced as a second postulate that the velocity of light,V, in empty space is an absolute constant of nature. Hence this velocity, contrary to all others, is independent of the motion of the body emitting and the one receiving the light. The startling nature of this postulate is readily understood when we remember that the closest analogue to light is sound, the velocity of which is known by experience to be dependent on these quantities.

He has recorded for us, that he became convinced of the necessity for these postulates, because the theory of electrodynamics developed by Maxwell leads to an asymmetry, when applied to moving bodies, which is not true experimentally. For instance, when a magnet and an electric conductor are moved with respect to each other, the phenomena of the forces developed are observed to depend only on the relative motion of the magnet and the conductor, but Maxwell's theory requires a different explanation according to which is moved and which remains at rest. Besides this discrepancy between observation and theory, experiments, devised with sufficient accuracy, fail to show any effect of the earth's motion through space on the phenomena of light.

It will be convenient to grant these postulates and to follow Professor Einstein's deductions before attempting any criticism. In the first place, their adoption does away with the possibility of an ether and revives the conception of space as a vacuum. Both theory and experiment show that an ether must be in absolute rest with respect to the motion of the earth, and the first postulate denies the possibility of absolute rest and motion. Again, the failure of Newtonian mechanics lies in the assumption, which is always tacitly made, that moving bodies are subject to the same mechanical laws as those at rest. As I have already pointed out, the modification to be made in the laws for ordinary bodies is very minute and would probably never have become of importance if modern theory had not been interested in the properties of bodies moving supposedly with velocities approximating that of light.

The root of the error in the mechanics of moving bodies, Professor Einstein believes, lies in our determination of time, and clear thinking in regard to time is nearly all that is necessary to clear up the trouble. Suppose a body, or a material point to be at rest relatively to a coordinate system of three rectangular lines, then its position can easily be determined by the ordinary geometrical method of measuring its distance from each of the lines by rigid measuring-rods. But if the point is in motion with respect to the reference system, its position depends on time and cannot be determined by the geometrical method. Our idea of time is usually defined by what we call isochronism; we say an event occurs at seven o'clock, when the occurrence of the event and the position of the hour hand of a clock at seven are simultaneous. But suppose the clock were at some distance from us, then we could observe the hand to be at seven only after it had passed beyond that figure, since it would take some time by any method of transmission for the intelligence to reach us. As the velocity of light is the greatest of all known motions, the least discrepancy would be caused by using light signals as the mode of transmitting such intelligence. Also if we accept the second postulate, our intelligence will be still more accurate because the velocity of light is unaffected by other motions and we thus avoid the difficulty caused by the question whether any relative motion between us and the clock is an approach or a separation. To illustrate further our confusion as regards the measurement of time, suppose two persons wish to record two events, which we shall grant to be simultaneous, occurring at different places, A and B. An observer at A records the event when it occurs at A as being at T_A time, and instantly signals the fact to an observer at B. Whatever the means of signalling, a certain time will elapse before the observer at B is cognizant of it. For the reasons given, we shall adopt light signals as the best method. B observes the signal as T_B time by a clock placed at B and immediately reflects it back to A, who receives it at T'_A time by his clock. Although we have granted that both events were simultaneous, it is evident they will not be so recorded by the two observers. B will record the event at A as being later than the one at B. But if the velocity of light be absolutely independent of all conditions, then we should find that the differences of time going and returning are the same, or $T_B - T_A = T'_A - T_B$. This relation, if satisfied by any two events occurring at a distance from each other, is Professor Einstein's definition of simultaneous events or isochronism. The definition may be put in this form: Two events are simultaneous, if the difference of time to flash a signal there and back is equal to twice the distance between the two positions divided by the velocity of light, V.

Not only does this limitation in our inability to measure time affect our ideas of time, but it also has an important and unsuspected influence on our ideas of the size and shape of a body. This may be shown by an example. Suppose we wish to measure a rigid rod of length, l, moving in the direction of its axis with a velocity, v, then there are two methods of measuring this length and they do not give concordant results.

First an observer may move with the rod and measure its length by applying to it a measuring-rod. He will evidently obtain the same result as if both he and the rod were at rest.

A second method is possible and is frequently employed. An observer remains at rest and notes the positions of the two ends of the rod, at a certain time, *t,* which he determines by means of clocks, also at rest and tested for synchronism. These two points are dependent on our ability to record simultaneous events; if the rod were at rest the problem would be the one already discussed, but in this case the rod has a velocity, v, and consequently the time used in signalling in one direction involves, $V - v$, and in the other $V + v$. So when he measures the distance between the two points, which may also be called the length of the rod, he will find it not equal to l as determined by the first method, if he still regards his clocks as isochronous; or if he determined two positions whose measured distance is l then his clocks will not be isochronous.

Now our measurements of length are usually made under the condition that we, clocks, and rods are all moving with the earth and so relatively at rest. Our measurements of length of bodies on the earth are thus different from those made on the same bodies by an observer stationed with his clocks in the moon. Also if an object on the earth is measured while it is moving with respect to the observer, it will have a different length from that which it has when relatively at rest. Thus the dimensions of a body are dependent on its velocity, and Newtonian mechanics, which assumes the contrary, must be modified when applied to bodies in motion.

Professor Einstein then derives a set of equations which will express the dimensions of a moving body as they appear to a stationary observer. These show that the length of every body moving with a velocity, v, is diminished in the direction of its motion by the fraction

$$1/\sqrt{1-(v/V)^2},$$

and its dimensions at right angles to its motion are unchanged. Thus a sphere in motion becomes an ellipsoid flattened in the direction of its motion by an amount equal to the above fraction. It is proper to say, that this effect is quite inappreciable at ordinary velocities. This is true even of so great a velocity as that of the earth around the sun. This velocity is about thirty kilometers per second while V is 300,000 kilometers per second. One diameter of the earth would thus, to an observer in the sun, appear shortened about 7.5 centimeters, or three inches. But this effect becomes quite important for velocities approaching one-tenth of V. And a velocity equal to that of light is absolutely unattainable because the length of the moving body would then be reduced to zero. So we have the curious anomaly of a finite velocity producing an infinite effect.

The same equations also show that if one of two clocks, which are synchronous when at rest, be moved with a velocity, v, the stationary clock will run faster each second than the moving one. The maximum value of the difference is when one clock runs infinitely faster than the other.

We must not lose sight of the fact that this discussion, so far as mechanical bodies and motions are concerned, is purely academic, because for them the ratio v/V is too small in any known case to have an appreciable effect. The principle of relativity has its greatest significance when applied to problems of electricity, provided we accept the theory of electrons. If the atom of matter be composed of particles of electricity, if the inertia of matter be variable and due to electrical reactions, and if the velocity of the electron approaches that of light, then the effect of its velocity on its shape, size and mass is an important matter. For, let *m* be the mass of an electron at rest, from Professor Einstein's formulae, its apparent mass in the direction of motion is

$$\frac{m}{\sqrt{1-\left(\frac{v}{V}\right)^2}},$$

and its mass at right angles to the motion which would make it resist change of direction is

$$\frac{m}{1-\left(\frac{v}{V}\right)^2}.$$

Thus we have the unusual result of not only a variable mass, but also a difference in mass according to the direction considered. There is a close analogy between this apparent increase in the inertia of a moving electrified body and the increase in the inertia of a body moving in a fluid. In the hydrodynamic problem, we do not find it necessary to consider the real mass of the body as variable, since we can attribute its apparent variation to the energy required to set the fluid in motion. We may, in a similar manner, for the electrodynamic problem, ascribe the apparent increase in mass to the energy necessary to overcome the reactions of the electromagnetic field through which the body moves, and thus keep the mechanical mass constant.

Another consequence of the hypothesis is that mass also depends on energy. This result is of extraordinary importance, for it means we have no ability to distinguish between the inertia of a physical system of bodies and its energy; in other words, between the inertia of a body and the energy content of the space surrounding it. When we recall Professor Einstein's conception of space and energy, which I have quoted, most persons will become convinced that no idea of the nature of matter could be more abstract. The mass of a single body thus remains constant only when its energy remains constant. If it gives out heat, light, or electromagnetic energy, or if it even moves, its mass continually decreases, until, theoretically at least, it would melt away into a complex of energy. I know of no name to designate so immaterial a thing as this complex of energy, located somewhere in vacuous space. To be sure the interchange between mass and energy is very slow, so slow that ordinary minds grow skeptical of its existence. Thus a body radiating enough heat to warm a kilogram of water, one degree Centigrade, would decrease in mass only about 4.6×10^{-11} grams. It is fortunate that mass dissipates so slowly or we should indeed be things dreams are made of.

The last deduction I shall note is that the temperature of a moving body is less when measured by a moving observer than by one who is stationary.

Such are some of the more important conclusions which have been derived from the principle of relativity. If they can be established and men can be persuaded that the universe is, or even may be, as they indicate, a revolution has been accomplished in scientific thought. The postulates are apparently so simple and at the same time so bold, the conclusions are deduced by such rigorous mathematical development, that we must give the highest praise to this memoir in which so many of the perplexities of modern science are attacked. If we are to criticise the conclusions of this hypothesis, and they are opposed to what seems reasonable, it must be done by examining the postulates; in them, as in all hypotheses, lies the real strength or weakness of the system.

Professor Einstein asks us to accept two postulates. The first is: We have no ability in us to determine absolute rest or motion. We can say only that one body is at rest or changes position with respect to others. Phenomena are conditioned entirely by the relative positions of bodies and we can gain nothing toward this explanation by introducing the idea of absolute position. The second postulate is: The velocity of light, V, is a universal and absolute constant.

Apparently no objection has been made to his statement that these two postulates are sufficient from which to derive all the conclusions mentioned. But it seems to me that at least two more independent postulates are advanced in the memoir on relativity, which must also be granted. I shall propose as a third, Professor Einstein's definition of time; and as a fourth, the assumption of the atomic nature of electricity. The fourth postulate carries with it as corollaries, that the amount of electricity per atom, or the electron as it is called, is a universal constant, and that the mass of the electron is variable. My excuse for offering them is, they cannot be derived from his postulates, and his conclusions require them.

The first postulate can be granted at once and unreservedly for all purely mechanical motions of bodies. These involve merely changes of position and do not affect the nature of bodies nor their phenomena. But it must be accepted with limitations when in addition to mechanical motion (and by mechanical motion I mean a mere change of position) one body is emitting energy in the form of heat, light or electricity, and the other receiving it. True relativity requires a strictly symmetrical arrangement. Now the mechanical motions of two bodies with respect to one another are symmetrical. If a body has a velocity, v_1, to the right and another a velocity, v_2, to the left, they approach each other with a velocity, $v_1 + v_2$. The same result is obtained if the velocities are interchanged. This symmetry is not the case if the first body is emitting energy and the second receiving it. The behavior of each is then conditioned by the behavior of the other and in an asymmetrical manner. As an illustration: Let a body, by a periodic disturbance of its parts send out a wave of heat; this proceeds, according to theory, through space in all directions and on encountering a second body produces a periodic disturbance in it. The effect of this energy is twofold; the motions of the parts of the two bodies are symmetrical but the wave itself does not proceed from the second body in all directions but only in the one already impressed upon it by the first body. To make this clearer, we may start a wave along a stretched string by plucking aside one part of it; the wave will travel along the string in two direction, causing successive parts to vibrate symmetrically, but these parts will send the wave in one direction only. So we may say the velocities of two bodies are relative because the quantities appear in the mathematical expression in a symmetrical manner; so also are their momenta or their masses multiplied by their velocities. But this is not the case for energy since it involves the square of the velocity. The squared velocity remains positive although we change direction and the composition of two energies is always an addition. For true relativity, a change in direction must be accompanied by a change in sign. The extension of the idea of relativity to involve cases of radiant energy is partly responsible for the conclusion that mass is a function of energy.

Professor Einstein's second postulate has been widely discussed and many attempts have been made to interpret it. At first sight it seems absurd to say that the motion of a body emitting light does not affect the velocity of the light emitted. But the fact remains that all our experiments, and they have been most accurate and searching, fail to find any difference in the velocity of light whether the sources and the recipient are at rest or in motion. As we have seen, Professor Einstein, with admirable directness, goes straight to the point. He accepts the reliability of these experiments; assumes the constancy of V as a postulate, determines its effect on the space-dimensions of bodies and on time, and finds that we must change our previous ideas of these radically. Let us, when discussing V, consider it under two heads: first, when light passes through transparent material bodies and is subject to experimental verification; secondly, when it passes through space absolutely deprived of material bodies and so is not subject to experimental evidence.

In the first case, we know that there is a true path and that light moves very approximately in straight lines, and we have quite accurately measured the time light requires to move from one place to another through various substances. V is here the length of path divided by the time —a true velocity. We know that this V is a variable; it is less in water than it is in air, and still less in glass. It is also less in dense air than in rarefied air. Not only does V vary with the kind of matter through which light passes, but it also depends on the motion of the medium as shown in the experiments on the velocity of light passing through columns of moving water made by Fizeau and repeated by Professor Michelson. The velocity of light in material media is therefore subject to the variations which influence the velocity of sound and other types of motion and evidently cannot be the V assumed by Professor Einstein to be an absolute constant.

He must then limit his postulate strictly to what is called the velocity of light in absolutely immaterial space. There are two methods of obtaining this value. We observe the difference in time between the calculated eclipse of some satellite and the recorded observation of the event or we use other stellar phenomena. If we know the distance and the time, we say the velocity of light in empty space is the quotient of the two. Such observations are quite inadequate to settle experimentally the question of the constancy of V. Our measurements of the distances are crude for such a purpose, and also the light must travel part of the way through a material such as the air. Neglecting such astronomical methods, we have left only determinations of V made on the earth where the path is necessarily through matter. Here, what we are to call the constant V can be found only by extrapolation. For example, we find that the ratio of the velocity in air and in water is about four to three and agrees with their refractive indices. We find also that the velocity in different gases as they are reduced in density, tends to a common value, which is independent of the kind of gas. If we take this extrapolated value of 3×10^{10} centimeters for the velocity of light in empty space and assume the refractive index of space to be one, then we may calculate back and by this method we find the absolute refractive index of air at ordinary pressure to be 1.0002. Such an experimental method and such reasoning are highly unsatisfactory. In the first place, we start with measurements of lengths and time which are subject to experimental verification and calculate V; from these we pass to the supposititious V where only one factor, the time, is subject to experimental proof and the other factor, the length, is not, as we cannot experiment on the path of light in free space. But passing this objection, we have calculated from length and time determinations a value for V and Professor Einstein declares it to be an absolute constant. He then reasons backward that the length of a body and time determinations must be variables with motion, in order to keep V constant. Of course, if Professor Einstein wishes to look on the universe as a purely abstract conception, and if he wishes to make what we sometimes call concrete or objective phenomena correspond to his preconceived idea, he *can* assume V in a vacuum to be a universal and absolute constant and no one can disprove it scientifically. But there are some who still cling to the idea that dimensions of bodies and time are not subject to our fancy, and who believe that if any quantities must vary, it should be those which we have no means of determining directly. And after all it is asking a great deal of us, to upset our ideas in order to explain at bottom a single experiment, that of Michelson and Morley, however accurately it has been performed and however puzzling its results may be. He has not even the justification his predecessors would have had. When the ether was believed to be a crystalline solid which vibrated with the passage of light rays, V had a real meaning although we could not determine it directly. When the ether was a substance which periodically varied electromagnetically, we could still say that V had a possible meaning. But the latest definition, that the ether is absolutely quiescent space to be distinguished from vacuous space only by the fact that it is the seat of an entity, called electromagnetic energy, and contains a light vector, makes the word velocity when applied to V, absolutely without meaning in any ordinary sense of the term. To speak of a motion in an absolutely quiescent space is a rather startling statement. The ether also becomes a local affair shifting back and forth in vacuous space according as light is present or not. The feeble light of a candle, apparently, changes nothing into something.

But aside from these considerations which lie in the debatable land and which will be decided largely by the temperament of the individual, there seems to be an absolute contradiction between the first two postulates as I understand them. If V be the value of the velocity of light in an absolutely quiescent ether and itself a constant, then the velocity of light in a material medium, such as air, is an absolute velocity or motion when referred to V. Now the first postulate declares that we can have no knowledge of absolute motion and, in addition, that the explanation of phenomena are not conditioned by absolute motion.

The third postulate, which I have proposed as necessary for the system proposed by Professor Einstein, is his definition of time. He says that our time is defined by synchronism or the simultaneous occurrence of an event and the position of the hour hand of a clock at a certain position, the number seven for example. This is certainly not our idea of time but merely our method of measuring it quantitatively. If we did not have an adequate idea of time as the mere succession of events we should have no conception of what simultaneous occurrences are. His definition gives me the feeling that if I could make clocks go slower my life would become longer. In the review of his memoir, it was pointed out that his definition of simultaneity or isochronism, when combined with the constancy of V, led to the conclusion that two clocks which were synchronous when relatively at rest, would not remain so if one of them were given a velocity. Moreover the length of a body in motion decreases with respect to its length when at rest. It is certain that these results depend on the constancy of V. If we are willing to rest our knowledge of time and space on a value which by the nature of things can never be put to a direct test, I am convinced of the theoretical correctness of these results. But I am not willing to do so. I prefer to trust to the invariability of time and space phenomena, even if it requires V to be a variable, and time measurements to contain an unavoidable discrepancy. By so doing, I am aware that I may forfeit an explanation of the Michelson-Morley effect and of some others, but this loss is not overwhelming and we can afford to wait for an explanation until a future time.

I am the more willing to take this attitude, because I can see no means of applying the principle of relativity to the test of experience. As we have seen, the velocity of light in any material medium is a variable and can be no more trusted than the velocity of sound.

Perhaps this illustration may make my meaning clearer. Suppose a race of men to exist who are blind and have no knowledge of electromagnetic radiation, but who wish to measure the lengths of moving bodies. They will undoubtedly be compelled to get this information and that of the synchronism of clocks by sound signals. It is evident, that observations carried out, under conditions similar to those imposed by Professor Einstein, would indicate that the length of a moving body underwent changes. And while they could make corrections for some of the effects, because sound waves are largely affected by the motion of media and of sonorous bodies, yet they would undoubtedly come to the conclusion that the dimensions of a moving body depended to some extent on its motion. Now, if we bestow sight on one of these men, he would be able to correct their measurements, because by his immensely more rapid light signals he could gain a much more nearly instantaneous value for synchronism. We are, at present, in the condition of this man. As we improve in our ability to measure the velocity of light under different conditions we shall, Professor Einstein thinks, get closer to the knowledge of the absolute V and to the relations for space and time which he has derived. But we may suppose men will some day find a kind of radiation which has a velocity greater than V and by its aid remove the conviction remaining in our minds that motion affects length and time. Calculation may show that material bodies cannot attain this velocity, but we are speaking of an immaterial radiation. To say that such a radiation is impossible is as futile, at least as unscientific, as for a race of the blind to say that there is no light; especially so, as it is possible that gravitational attraction may involve a velocity of transmission vastly greater than that of light.

Since the motion of any ponderable body is too slow to make the ratio v/V an appreciable quantity, the only supposable case where this ratio can enter as a determining factor is in problems of radioactivity and the discharge of electricity through gases. In these, the particles of matter are supposed to be so small and to have a velocity so great that their mass and size are functions of their velocity. But to limit the applicability of the principle of relativity to such supposititious bodies as electrons, is to rob it of much of its importance, and we should hardly consider it one of the great principles of nature.

But even this is not all. If we wish to apply the principle to electrons, we shall need a fourth postulate. We must assume that a quantity of electricity is atomic in character; either the charge of electricity associated with what we call an atom of matter is an invariable, or, as it is now usually expressed, electricity is an entity which may be divided into invariable, equal, and indivisible parts called electrons. As a corollary to this postulate, the ponderable mass of an atom is a variable depending on its velocity. It will suffice to consider this postulate very briefly. It may be shown at once that the constancy of electric quantity and the variability of mass is not a necessary assumption. All experiments, which involve both these quantities, include them in the form of a simple ratio, e/m; where e is the quantity of electricity and m the mass of the electron. It is evident that any value may be given to this ratio by supposing either one of its members to remain constant and the other to vary, or by supposing both to vary in opposite fashions, and it may be shown that it is just as reasonable to keep m constant and to let e vary with the velocity as to make the contrary assumption. This is certainly possible until we have experimental evidence which will determine our decision and this evidence is not likely ever to be at our disposal.

If we attempt to estimate the results which follow from the postulates of relativity, the first would be that Professor Einstein has proved that we cannot theoretically measure in space and time moving bodies exactly by the laws which apply to bodies at rest. The first postulate will be accepted for all mechanical motions and no further attempts will be made to find experimentally an absolute motion. But the second postulate contradicts the principle of relativity, if by V we mean the absolute motion of *something*—matter, energy, or light through quiescent space. If, on the other hand, V is understood to be the velocity of light in space containing matter, then it cannot be taken as a universal constant. The third postulate concerning time should not cause us to change our belief that the dimensions of a body and the unit of time are independent of velocity, but it should show us how to correct our measurements of moving bodies, as we must correct all subjective measurements of objective phenomena.

If we grant the assumptions of Professor Einstein, the theory of relativity is a perfectly logical system so long as we apply it to abstract systems moving with a constant velocity in a straight line. But I can find no evidence that the postulates agree with experimental facts and certainly, to the present time at least, its conclusions will not affect experimentally the laws of mechanics. Also its theoretical conclusions are limited, as yet, to the rare cases when there is neither any change of speed nor of direction. We should remember before we abandon or modify Newtonian mechanics that mathematicians have devised numerous systems which point to different mechanical laws. If we postulate a fourth dimension to space we can deduce perfectly logical laws for a mechanical system that is quite different from the laws of ordinary mechanics. And the universe, constructed on this principle, is a very beautiful and interesting one. It is the privilege of mathematicians to deal with symbols; the physical universe is no more important to them than any other universe which can be developed symbolically. The trouble occurs when the distinction between the real or physical universe and the symbolical or metaphysical universe is obliterated in the minds of men of science.

LOUIS TRENCHARD MORE.
UNIVERSITY OF CINCINNATI.